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MEASUREMENT OF VISCOPLASTIC PROPERTIES OF A LIQUID IN EXPERIMENTS WITH A TORSION VISCOSIMETER

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The results of a numerical solution of the problem of vibrations of a torsion viscosimeter filled with an incompressible viscoplastic liquid are presented. It has been shown that in the vicinity of the rotation axis there appears a dead zone, whose boundary changes in the process of vibrations. The influence of plastic properties of the liquid on the characteristics of viscosimeter vibrations has been determined. A method for identifying viscoplastic properties by the observed parameters of vibrations is proposed.

The method of torsional vibrations is widely used in practice and, in particular, in investigations of internal friction in condensed media. In the physics of liquids, especially aggressive ones, it is one of the main techniques of measuring viscosity. Its chief advantage is the possibility of recording the observed parameters (period and damping decrement of vibrations) with an accuracy hardly possible or impossible in other methods. This makes it possible to realize it as an absolute method if one manages to solve the problem on the relation between the observed parameters and the liquid parameters with a fairly high degree of accuracy. Such a problem has been solved analytically only for Newtonian [1] or viscoelastic liquids [2]. The potentialities of the method for investigating the rheological properties of other liquids are still not clearly understood. The present paper attempts to estimate them for liquids called viscoplastic liquids, whose flow becomes possible only after the shear stress crosses some threshold — yield point. Taking into consideration the known mathematical problems in describing even simple flows of such liquids [3], we restrict ourselves to the investigation of some idealized numerical model of viscosimeter motion which, nevertheless, enables us to answer the question about the features of this motion determined by the difference of the considered medium from a Newtonian liquid.

Mathematical Model. Let a cylindrical vessel be filled with a viscoplastic liquid and execute damping torsional vibrations about its axis being suspended by an elastic fiber. The task is to determine the influence of the viscoplastic properties on the frequency f and the damping coefficient p of cylinder vibrations. In the approximation of an infinitely long cylinder and on the assumption of axial symmetry of the flow, only the azimuthal velocity component of the liquid V_φ is other than zero. The equation of motion of a viscoplastic medium in the viscous flow zone and the constitutive equations for linear motion of a viscoplastic material will be written as formulas (1) and (2) in [4]. It is clear that in the approximation used the dead zone may represent either a cylinder (if it is at the center of the viscosimeter) or a cylindrical layer of some thickness. Then the equations of motion of the viscosimeter cylinder and dead zone will be written as

$$I_{\mathrm{c}} \frac{d\omega_{\mathrm{c}}}{dt} = -\kappa\theta + M_{\mathrm{c}} , \tag{1}$$

$$I_i \frac{d\omega_i}{dt} = M_i . \tag{2}$$

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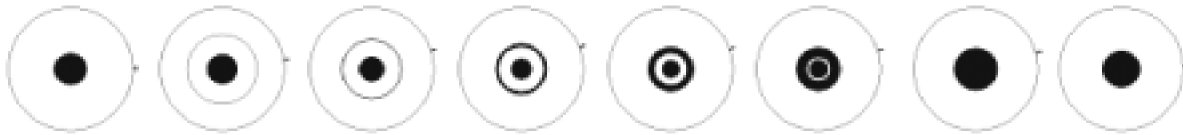


Fig. 1. Distribution of dead zones in the viscosimeter cross-section over the first half-period of the fifth vibration. Mark on the external circle denotes the deviation from the equilibrium position.

In (2), it is assumed that several dead zones can exist simultaneously, i is the ordinal number of a zone counting from the axis of the viscosimeter cylinder. As boundary conditions, the conditions of adhesion on solid boundaries are taken:

$$V_\varphi (R_c, t) = \omega_c R_c \,, \quad V_\varphi (R_{ex,i}, t) = \omega_i R_{ex,i} \,, \quad V_\varphi (R_{in,i}, t) = \omega_i R_{in,i} \,. \tag{3}$$

Before beginning to move, the cylinder together with the liquid is at rest in a position turned by some angle θ_0 with respect to the equilibrium position and at time $t = 0$ the cylinder is released. The initial conditions in this case are written as

$$V_\varphi (r, 0) = 0 \,, \quad \theta (0) = \theta_0 \,, \quad \omega_c (0) = 0 \,. \tag{4}$$

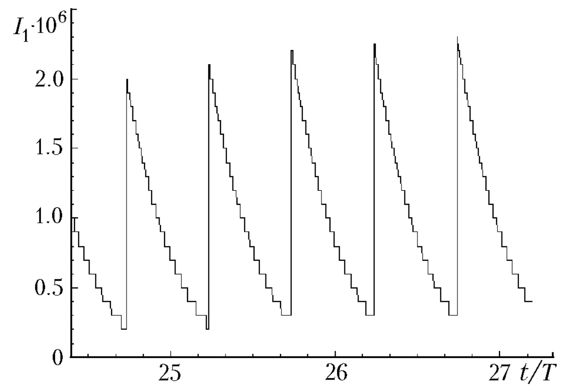
Numerical Solution. We seek it by the finite difference method. The equations of motion of the viscoplastic medium in the viscous flow zone, the constitutive equations for the viscoplastic material, as well as expressions (1)–(4) are reduced to dimensionless form so that distances are related to the radius R_c, velocities — to ν/R_c, pressure — to $\rho\nu^2/R_c^2$, and time — to R_c^2/ν. A uniform grid with a maximum discretization of up to 2000 in the radial direction is used. The derivatives are discretized and the difference equations are solved in much the same way as in [4]. In solving the system of difference equations at time t^{n+1}, the solid-viscous position of the boundary determined at the previous time step is used. The value of δ corresponding to the accuracy of determination of the deformation rate in the difference scheme was determined as in [4]: $\delta = \tau_0 R_c^2/(N\nu^2\rho)$.

Results and discussion. At the initial instant of time the deformation rate is other than zero only in the region immediately adjoining the cylinder. Over the first period of vibrations the radius of the dead zone located along the cylinder decreases. If the initial deviation angle is small enough (<0.05 rad), then complete "dispersal" of the axial dead zone in the process of vibrations does not occur. The evolution of dead zones in the viscosimeter cross-section over the first half-period of the fifth vibration is shown in Fig. 1. As is seen, at the moment of passing through the equilibrium position in the cylinder only one axial zone exists. As the viscosimeter decelerates, a ring-shaped zone is formed and then the axial and the ring-shaped dead zone approach each other and "coalesce." Moreover, there is a gradual increase in the maximum radius of the axial dead zone, which affects the time dependence of its moment of inertia (Fig. 2). This dependence has a complex nonharmonic shape and a double frequency as compared to the viscosimeter frequency, since the fulfillment of the conditions of solid-viscous separation is not associated with the sign of stresses [4] and, therefore, is reproduced twice over one period. As is seen from Fig. 2, the mean moment of inertia increases in the process of vibrations, which is due to the decrease in the amplitudes of liquid velocities inside the viscosimeter cylinder as a result of damping and, consequently, to the expansion, on average, of the region of motion of a rigid body. The periodic stepwise increase in the moment of inertia in Fig. 2 is due to the "coalescence" of the axial and ring-shaped zones.

Figure 3 presents the dependence of the damping coefficient of viscosimeter vobrations on the vibration number for various values of the yield point τ_0. To obtain these curves, one has to adjust the law of motion of the viscosimeter $\theta(t)$ to a function of the form

$$\theta (t) = A \exp (-pt) \sin (ft + \psi) \tag{5}$$

in order to determine the vibration frequency f and the damping coefficient p. Adjustment was carried out by the least square technique with minimization by the Rosenbrock method [5].



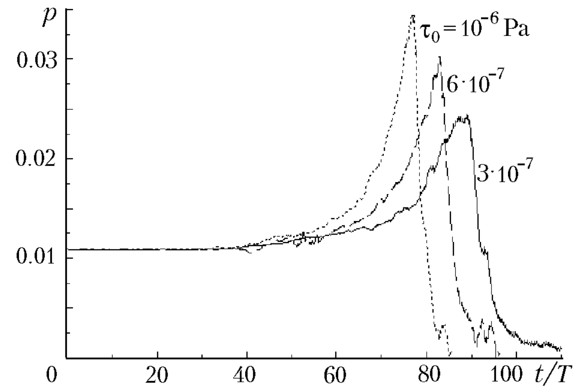


Fig. 2. Moment of inertia of the axial dead zone as a function of the vibration number: $R_c = 0.01$ m, $\tau_0 = 10^{-6}$, Pa. I_1, kg·m^2.

Fig. 3. Locally determined damping coefficient as a function of the vibration number for liquids with different yield points. p, sec^{-1}.

Measurements of vibration characteristics for Fig. 3 were made locally on the basis of the data from a small part of the vibration record. As is seen from Fig. 3, the interval of stationary values of p is replaced by a segment in which there is an increase in the damping coefficient, which is likely to be due to the fact that the boundary layers on the side surfaces of the viscosimeter and the axial dead zone begin to overlap strongly influencing each other. The interval of increase in the damping coefficient lasts for about 10–15 vibrations and, in so doing, it changes by a value that is large enough to be registered in experiments with a torsion viscosimeter, where the relative error of measurement of the damping coefficient reaches 10^{-4}. The next stage of vibration development is characterized by a decrease in the damping coefficient almost to zero. This is due to the fact that the radius of the axial dead zone increases so that it can "stick" to the cylinder and then the whole system moves as a rigid body. The decay thereby is determined by the scheme viscosity and is therefore small. As can be seen from Fig. 3, the change in the character of vibrations is the faster, the larger the value of the yield point. Thus, all other things being equal, for the yield point $\tau_0 = 1.0 \cdot 10^{-6}$ Pa the maximum of p falls on the 77th vibration, whereas for $\tau_0 = 6.0 \cdot 10^{-7}$ Pa it falls on the 83d vibration. This fact can be used to estimate the plastic properties of a liquid — the yield point τ_0. Note that such viscoplastic liquids as biological ones (blood, blood plasma, etc.) have yield points of $\approx 10^{-3}$ Pa [6], which is three orders of magnitude higher than the detection limits of the method of torsional vibrations ($\approx 10^{-6}$ Pa). This means that it is possible to investigate media with much less pronounced viscoplastic properties than even in biological liquids.

The experimental law of viscosimeter motion usually contains noise and systematic errors. Therefore, in adjusting it to function (5) it is recommended to perform a local spectral analysis of the signal and average the data over one or several periods. Such a procedure will have a stronger effect on the shape of the maximum than on its position. Therefore, it is expected that this will not affect the accuracy of determination of the time at which a maximum of the function $p(t)$ appears.

Thus, the revealed features in the time dependence of the damping coefficient of vibrations of a torsion viscosimeter permit judging the viscoplastic properties of a liquid and can be the basis for determining its yield point.

NOTATION

A, vibration amplitude of the viscosimeter, rad; f, vibration frequency of the viscosimeter, sec^{-1}; I_i, moment of inertia of a dead zone, kg·m^2; I_c, moment of inertia of the viscosimeter, kg·m^2; M_i, moment of friction forces acting on a dead zone, N·m; M_c, moment of friction forces acting on the cylinder, N·m; N, number of spatial nodes of the difference scheme; p, damping coefficient of vibrations, sec^{-1}; $R_{ex,i}$, $R_{in,i}$, external and internal radii of the ith dead zone, m; R_c, internal radius of the viscosimeter cylinder, m; t, time, sec; t^n, dimensionless time of the difference scheme, nth step; T, period of viscosimeter vibrations, sec; V_φ, azimuthal component of the liquid velocity, m/sec; δ, dimensionless numerical criterion for determining the rigid body motion–viscous flow boundary; θ, rotation angle of the cylinder, rad; θ_0, initial angle of rotation of the cylinder, rad; κ, coefficient of torsional rigidity of the fiber, Pa; ν, kinematic viscosity coefficient, m^2/sec; ρ, liquid density, kg/m^3; τ_0, yield point of the liquid, Pa; ω_c, angular velocity of the cylinder, sec^{-1}; ω_i, angular velocity of the *i*th dead zone, sec^{-1}; ψ, vibration phase of the viscosimeter, rad. Subscripts: c, cylinder of the viscosimeter; ex, external; in, internal; *i*, ordinal number of a dead zone; *n*, step number of the difference scheme.

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